AP Calculus AB Syllabus

Jeffrey Z. Lawson

Hugh M. Cummings High School

Burlington, NC

Overview

AP Calculus AB is comparable to a calculus course in a college or university. Students are expected to seek college credit and/or college placement from colleges or universities upon completion of the course. The AP Program specifies the scope of the course and the parameters for the exam.

Goals

- To work with functions represented in a variety of ways: graphically, numerically, analytically, and verbally as well as to understand the connections among these representations.
- To understand the derivative in terms of a rate of change and a local linear approximation and to use derivatives to solve a variety of problems
- To understand the definite integral both as a limit of Riemann sums and as the net accumulation of change and to use integrals to solve a variety of problems
- To understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus
- To communicate mathematics and explain solutions to problems both verbally and in written sentences
- To model a written description of a physical situation with a function, a differential equation, or an integral
- To use technology to help solve problems, experiment, interpret results, and support conclusions
- To determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement
- To develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment

Technology Use

Students use graphing calculators regularly, when appropriate, for discovery and reflection. In class students will use TI-84 Plus calculators to solve problems, experiment, interpret results, and support conclusions, as specified in the goals of the course.

A graphing calculator appropriate for use on the exams is expected to have the built-in capability to:

- Plot the graph of a function within an arbitrary viewing window

- Find the zeros of functions (solve equations numerically)
- Numerically calculate the derivative of a function
- Numerically calculate the value of a definite integral

Note that students must have the ability to use paper-and-pencil techniques of calculus and apply them when technological tools are unavailable or inappropriate.

Topic Outline

This topic outline indicates the scope of the course, although it does not reflect the order in which the topics are taught. The exam is based on the topics listed here, but additional topics may be included for enrichment. The times indicated are estimates and vary from year to year based on students' abilities and interests.

Part 1: Functions, Graphs, and Limits (25 days)

Analysis of Graphs

- With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process
- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

- An intuitive understanding of continuity (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

Part 2: Derivatives (55 days)

Concept of the derivative

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of the average rate of change
- Approximate rate of change from graphs and tables of values

Derivative as a function

- Corresponding characteristics of graphs of f and f'
- Relationship between the increasing and decreasing behavior of f and the sign of f'
- The Mean Value Theorem and its geometric interpretation
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of graphs of f, f', and f''
- Relationship between the concavity of f and the sign of f''
- Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity
- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse function
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Derivative rules for sums, products, and quotients of functions
- Chain rule and implicit differentiation

<u>Part 3: Integrals (50 days)</u>

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f'(x)\,dx = f(b) - f(a)$
- Basic properties of definite integrals (examples include additivity and linearity)

Applications of integrals

- Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions
- Antiderivatives by substitution of variables (including change of limits for definite integrals)

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line
- Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth)

Numerical approximations to definite integrals

- Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

Preparing for the AP Exam (20 days)

Students need to practice both multiple choice and free response questions to gain confidence and to experience the style of exam questions. Questions are taken from released versions of AP exams and are used throughout the course as classwork, homework, and test questions.

In April during the exam review preparation period, practice tests are given under examination conditions. This time is used to focus on topics that appear to need more attention and review. We spend this time strengthening communication skills, both oral and written, reteaching topics that need further input, and practicing for mastery of concepts.

Primary Textbook

Larson, Hostetler, Edwards. *Calculus of a Single Variable*. Sixth edition. Houghton Mifflin Company, 1998.

Additional Resources

Rogawski, Jon, Ray Cannon. *Calculus for AP*. Second edition. W. H. Freeman and Company, 2012

Finney, Ross L., Franklin D. Demana, Bert K. Watts, and Daniel Kennedy. *Calculus – Graphical, Numerical, Algebraic*. Third edition. Pearson, Prentice Hall, 2007.

Crawford, Debra, Mary Ann Gore, Jill Gough, and Sam Gough. *Work Smarter Not Harder (Calculus Activities for the TI-83/TI-84*). Venture Publishing Company.

AP Calculus Multiple Choice Question Collection 1969 – 1998

AP Calculus Free Response Questions and Solutions 1989 – 1997

References

apcentral.collegeboard.com

AP Calculus Workshop Handbook 2011-2012

AP Calculus AB and BC Course Description Effective Fall 2012

AP Calculus Teacher's Guide