AP Calculus BC Syllabus

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Overview

AP Calculus BC is an extension of AP Calculus AB including more topics for examination covered in rigorous depth. Students are expected to seek college credit and/or college placement from colleges or universities upon completion of the course. The content for Calculus BC is designed to qualify the student for placement and credit in a course that is one course beyond that granted for Calculus AB.

The AP Program specifies the scope of the course and the parameters for the exam. Themes related to limits, derivatives, integrals, approximations, and modeling and applications form the key to developing the course and using all the types of functions indicated by the College Board *Course Description* document.

Goals

- To work with functions represented in a variety of ways: graphically, numerically, analytically, and verbally as well as to understand the connections among these representations.
- To understand the derivative in terms of a rate of change and a local linear approximation and to use derivatives to solve a variety of problems
- To understand the definite integral both as a limit of Riemann sums and as the net accumulation of change and to use integrals to solve a variety of problems
- To understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus
- To communicate mathematics and explain solutions to problems both verbally and in written sentences
- To model a written description of a physical situation with a function, a differential equation, or an integral
- To use technology to help solve problems, experiment, interpret results, and support conclusions
- To determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement
- To develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment

Technology Use

Students use graphing calculators regularly, when appropriate, for discovery and reflection. In class students will use TI-84 Plus C calculators to solve problems, experiment, interpret results, and support conclusions, as specified in the goals of the course.

A graphing calculator appropriate for use on the exams is expected to have the built-in capability to:

- Plot the graph of a function within an arbitrary viewing window
- Find the zeros of functions (solve equations numerically)
- Numerically calculate the derivative of a function
- Numerically calculate the value of a definite integral

Note that students must have the ability to use paper-and-pencil techniques of calculus and apply them when technological tools are unavailable or inappropriate. Regular attention is given to determining when and how technology is appropriate for use.

Topic Outline

This topic outline indicates the scope of the course, although it does not reflect the order in which the topics are taught. The exam is based on the topics listed here, but additional topics may be included for enrichment. The times indicated are estimates and vary from year to year based on students' abilities and interests.

Part 1: Functions, Graphs, and Limits (20 days)

Analysis of Graphs

- With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process
- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity

- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

- An intuitive understanding of continuity (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

Parametric, polar, and vector functions

- An analysis of planar curves given in parametric form, polar form, and vector form

Part 2: Derivatives (45 days)

Concept of the derivative

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of the average rate of change
- Approximate rate of change from graphs and tables of values

Derivative as a function

- Corresponding characteristics of graphs of f and f'
- Relationship between the increasing and decreasing behavior of f and the sign of f'
- The Mean Value Theorem and its geometric interpretation
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of graphs of f , f', and f''
- Relationship between the concavity of f and the sign of f''
- Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity
- Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration
- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse function
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations
- Numerical solution of differential equations using Euler's method
- L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Derivative rules for sums, products, and quotients of functions
- Chain rule and implicit differentiation
- Derivatives of parametric, polar, and vector functions

<u>Part 3: Integrals (45 days)</u>

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f'(x)\,dx = f(b) - f(a)$
- Basic properties of definite integrals (examples include additivity and linearity)

Applications of integrals

- Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve (including a curve given in parametric form), and accumulated change from a rate of change.

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions
- Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only)
- Improper integrals (as limits of definite integrals)

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line
- Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth)
- Solving logistic differential equations and using them in modeling

Numerical approximations to definite integrals

- Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

<u>Part 4: Polynomial Approximations and Series (26 days)</u>

Concept of series

- A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence and divergence.

Series of constants

- Motivating examples, including decimal expansion
- Geometric series with applications
- The harmonic series
- Alternating series with error bound

- Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series
- The ratio test for convergence and divergence
- Comparing series to test for convergence and divergence

Taylor series

- Taylor polynomial approximation with graphical demonstration of convergence, *e.g.* viewing graphs of various Taylor polynomials of the sine function approximating a sine curve
- Maclaurin series and the general Taylor series centered at *x = a*
- Maclaurin series for the functions e^x, $\sin x$, $\cos x$, $\frac{1}{1-x}$
- Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series
- Functions defined by power series
- Radius and interval of convergence of power series
- Lagrange error bound for Taylor polynomials

Classroom Practices

This course teaches students to explain problems orally.

Students are required to do regular board work, using exercises completed for homework as well as class starters, where they explain to the class their process and rationale leading toward solutions. Students are then required to answer questions put to them by their classmates.

Students work in cooperative pairs and/or groups on a regular basis. In their pairs/groups they are asked to discuss with others the how they might approach a problem and why they would use the particular approach. Students are encouraged to contribute what they know and to ask questions about what they do not know.

Students have project work each quarter in which they are required to explain to classmates, using proper mathematical language, the underlying Calculus concepts involved in reaching a solution to the project problem.

Students work released free-response questions from past AP Exams and are required to explain to the class as a whole as well as to small groups how they reached their conclusions/solutions. Focus on proper mathematical language, precision and accuracy of justifications, and thoroughness are stressed.

This course teaches students to explain problems in written sentences.

Students do regular writing assignments where they are required to write the explanations to processes or procedures, to explain Calculus concepts, or to explore, surmise, and extrapolate about specific real-world problems that may be solved using Calculus.

Students have project work each quarter in which they are required to write, using proper mathematical language, explanations of mathematical concepts and specifically, the underlying Calculus concepts involved in the solutions.

Students work released free-response questions from past AP Exams and are required to write their responses which are then graded using the rubrics provided by the College Board. A focus on correctly writing justifications in an appropriate format is a top priority.

This course teaches students to use graphing calculators to experiment.

Students use graphing calculators to explore a variety of Calculus concepts including, but not limited to:

Investigating the numerical approach to evaluate the limit of a function

Determining the asymptotic nature of a function

Understanding the continuity of a function

Numerically exploring the value of a derivative at a point

Calculating the value of a definite integral

To predict and find the linearization of a function

To explore slope fields and predict solutions using them

This course teaches students to use graphing calculators to interpret results and support conclusions.

Students use graphing calculators to interpret results and support their conclusions through activities including, but not limited to:

Graphing functions and determining if they fit given characteristics

Graphing functions to determine if they are valid as solutions to specific problems

Graphing functions to analyze specific types of asymptotic behavior

Graphing functions to determine why a derivative fails to exist

Graphing functions to explore area under a curve and/or between two curves

Graphing functions to compare the effects of varying a parameter

Preparing for the AP Exam (16 days)

Students need to practice both multiple choice and free response questions to gain confidence and to experience the style of exam questions. Questions are taken from released versions of AP exams and are used throughout the course as classwork, homework, and test questions.

In April during the exam review preparation period, practice tests are given under examination conditions. This time is used to focus on topics that appear to need more attention and review. We spend this time strengthening communication skills, both oral and written, reteaching topics that need further input, and practicing for mastery of concepts.

In addition, time is spent working on correctly writing responses to free-response questions, paying particular attention to writing justifications, appropriate simplifications, rounding, and related details. Discussion about when and how to appropriately use a graphing utility is also reviewed in the process of working through practice exams.

Primary Textbook

Larson, Ron, Bruce Edwards. *Calculus*. Tenth edition. Brooks/Cole, Cengage Learning, 2014.

Additional Resources

Hughes-Hallett, Deborah, Andrew M. Gleason, William G. McCallum. *Calculus, Single and Multivariable*. Sixth edition. John Wiley & Sons, Inc., 2013.

Stewart, James. *Single Variable Calculus, Early Transcendentals*. Eighth edition. Cengage Learning, 2012.

Rogawski, Jon, Ray Cannon. *Calculus for AP*. Second edition. W. H. Freeman and Company, 2012.

Finney, Ross L., Franklin D. Demana, Bert K. Watts, and Daniel Kennedy. *Calculus – Graphical, Numerical, Algebraic*. Third edition. Pearson, Prentice Hall, 2007.

Crawford, Debra, Mary Ann Gore, Jill Gough, and Sam Gough. *Work Smarter Not Harder (Calculus*

Activities for the TI-83/TI-84). Venture Publishing Company.

AP Calculus Multiple Choice Question Collection 1969 – 1998

AP Calculus Free Response Questions and Solutions 1989 – 1997

References

apcentral.collegeboard.com

AP Calculus Workshop Handbook 2015-2016

AP Calculus AB and BC Course Description Effective Fall 2012

AP Calculus Teacher's Guide