Warm-Up 2/21/14

Find the expected outcome.

1. You choose a number from 000-999. You pay $3 and win $10 if you're right.

\[
\frac{1}{1000} (7) + \frac{999}{1000} (3) = -2.99
\]

\[
\frac{1}{1000} (10) \times .01 = .01
\]

\[
\frac{1}{1000} (10) \times 10 = 10
\]

2. You pay $2 to roll two 6-sided dice. You win $30 if you get a sum of 9.

\[
\frac{3,6}{36} + \frac{4}{36} + \frac{2}{36} + \frac{1}{36} + \frac{1}{36} + \frac{2}{36} = \frac{3,33}{36} = \frac{29.97}{3}
\]

3. Find the fair price for 1-2.

4. Find the fair prize for 1-2.
COMPLEMENTS, INTERSECTIONS, AND UNIONS

A complement is when an outcome does NOT happen. Subtract the outcomes from the total.

1. What is the probability of NOT getting a sum of 7 on two dice?
   \[
   \frac{36}{36} - \frac{6}{36} = \frac{30}{36} \equiv \frac{5}{6}
   \]

An intersection is when two outcomes happen at the same time. This uses the word AND.

\[ P(A \text{ and } B) = P(A) \cdot P(B) \]

2. What is the probability the first die is a 3 AND the second is odd?
   \[
   \frac{1}{6} \cdot \frac{3}{6} = \frac{3}{36} = \frac{1}{12}
   \]

3. What is the probability the first die is odd AND the sum is seven?
   \[
   \begin{array}{c}
   1, 6, 3, 4 \\
   6, 1, 4, 3 \\
   2, 5 \\
   5, 2
   \end{array}
   \]
   \[
   \frac{3}{36} = \frac{1}{12}
   \]
Drawing one card:

4. What is the probability the card is a spade AND a face card? $\frac{3}{52}$

5. What is the probability it is a red card AND a queen? $\frac{2}{52} = \frac{1}{26}$

6. A spade AND a heart?

$\frac{0}{52}$
A union is when at least one of the two outcomes happens (using the word OR).

Add each outcome together, and subtract the AND. \[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

Rolling two dice, find the probability that:

1. The first die is a 2 OR the second die is odd.
   \[ \frac{6}{36} + \frac{18}{36} - \frac{3}{36} = \frac{21}{36} \]

2. The sum of the dice is six OR the first die is even.
   \[ \frac{5}{36} + \frac{18}{36} - \frac{2}{36} = \frac{21}{36} \]

3. The first die is a 5 OR the sum of the dice is less than 5.
   \[ \frac{6}{36} + \frac{6}{36} - \frac{0}{36} = \frac{12}{36} = \frac{1}{3} \]
Drawing a card:

4. The card is a two OR a diamond?
\[
\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}
\]

5. The card is a 3 OR a face card?
\[
\frac{4}{52} + \frac{12}{52} - 0 = \frac{16}{52} = \frac{4}{13}
\]

6. The card is a diamond OR a red card?
\[
\frac{13}{52} + \frac{26}{52} - \frac{13}{52} = \frac{26}{52} = \frac{1}{2}
\]

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A
8-Complements, Intersections, and Unions

February 24, 2014

Dice and cards are shown with numbers and symbols, illustrating concepts in set theory.
SURVEY QUESTIONS

We give three separate surveys to the same ten people.

- 3 say they like hip hop.
- 5 say they like rock.
- 7 say they like hip hop OR rock (note: this means like they like at least one of the two)

How many people like both?

\[ \text{H + R - \ AND} = \ OR \]  
\[ \text{(both)} \ (at \ least \ one) \]

Then \[ \text{H + R - \ OR} = \ AND \]

\[ 3 + 5 - 7 = 1 \]

This is because \[ 3 + 5 - 1 = 7 \]

\[ H + R - \text{both} = \text{at least one} \]
Ex: Do you like dogs? Do you like cats?

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<th>D</th>
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5 dogs | 1 cats | 1 Both | 5 at least | 1

How many liked both if 5 liked at least 1.

5 + 1 - 5 = 1 ✓

at least 1.

5 + 1 - 1 = 5 ✓
CLASSWORK:

Come up with two choices. Ask five people if they like the two choices. Note: Someone can check either one, both of them, or neither.

Once you get your results, calculate how many like both, and how many like at least one.

Then, show that our formula works.

Ex:  Do you like dogs?  Do you like cats?

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