Taking a Spin

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Purpose

Although students are often asked to find the angles of rotational symmetry for given regular polygons, in this task they are asked to find the regular polygons for a given angle of rotational symmetry, a reversal that yields some surprising results. This task would be most appropriate with students who have at least some experience in exploring rotational symmetry.

Task Overview

What regular polygons have 80-degree rotational symmetry? Explain. *An activity sheet that gives students the complete task is included.*

Focus on Reasoning and Sense Making

Reasoning Habits

Focus in High School Mathematics: Reasoning and Sense Making

Analyzing a problem—identifying concepts, representations, or procedures; considering simpler analogs

Implementing a strategy—monitoring progress toward a solution

Reflecting on a solution—reconciling different approaches; justifying or validating; refining arguments; revisiting initial assumptions

Process Standards

Principles and Standards for School Mathematics

Problem solving—monitor and reflect on the process of mathematical problem solving

Reasoning and proof—select and use various types of reasoning and methods of proof

Connections—recognize and use connections among mathematical ideas

Standards for Mathematical Practice

Common Core State Standards for Mathematics

3. Construct viable arguments and critique the reasoning of others.

8. Look for and express regularity in repeated reasoning.

Focus on Mathematical Content

Key Elements

Focus in High School Mathematics: Reasoning and Sense Making

Reasoning with geometry—conjecturing about geometric objects; construction and evaluation of geometric arguments; multiple geometric approaches

Standards for Mathematical Content

Common Core State Standards for Mathematics

G-CO-3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-CO-4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Materials and Technology

- Taking a Spin activity sheet (optional)
- An online applet that demonstrates rotational symmetry might be useful—see for example http://www.MathRSM.net/applets/spin/. Alternatively, teachers could use pictures of regular polygons to illustrate the concept of rotational symmetry.

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Use in the Classroom

If your students have not recently worked with rotational symmetry, you might begin with a review of the topic. An online applet that demonstrates rotational symmetry might be useful—one such applet is available at http://www.MathRSM.net/applets/spin. Alternatively, you might use pictures of regular polygons to illustrate the concept.

Making connections to prior knowledge is an important aspect of analyzing a problem.

The students then might work on the task individually, either for homework or in class, and subsequently work in groups to discuss their answers.

Encouraging students to critique the reasoning of their classmates and reconcile the different approaches used is an important reasoning skill.

Students often suppose that no regular polygon satisfies the requirement, since 360 is not divisible by 80. You might ask, "Can anyone think of a polygon that you believe has 80-degree rotational symmetry? Can you convince us?"

Asking students to justify their findings enhances and strengthens their understanding.

If the students continue to be at a loss, discussing all the rotational symmetries of a regular triangle might help; again, working with an online applet could be useful.

Looking at a simpler analog can be a valuable tool for analyzing a problem situation.

Once students find one solution, you might ask them to look for other possibilities, either individually or in groups. As they find additional solutions, you could record them on the board. Have the students verify that the suggestions work, asking, "How do you know that this works?"

Students need to be constantly monitoring their progress toward a full solution and justifying their findings as they go.

When students think they have found all possible solutions, you might ask, "Have we really found them all? Might there be another one that we haven't considered?" To help your students focus on the pattern, a useful question might be, "What do all of these examples have in common?" (The following section provides further commentary on how students might productively think about this problem; this might provide further insights into productive lines of questioning.)

Reflecting on their findings can help students to see the regularities in their reasoning and refine their arguments about what regular polygons will work.

To conclude the discussion, you might ask, "Why did you initially think that you could not find a polygon that worked? What assumptions were you making? Why did it turn out that your assumptions were wrong?" You might also encourage students to prepare a more formal argument demonstrating that they have found all the solutions.

Students should be encouraged to revisit their initial assumptions to ensure understanding of their solution.

Focus on Student Thinking

Students might have seen that a regular n-gon has $360/n$ degrees of rotational symmetry, since n angles are formed from the center of the polygon to its vertices, and all of those angles are congruent. They may mistakenly apply this observation to conclude that no regular polygon can have 80-degree rotational symmetry, since the equation $360/n = 80$ does not have an integer solution.

To help your students examine this faulty assumption more deeply, you might ask, "Can anyone think of a different way that a regular polygon could have 80-degree rotational symmetry?" One of the easiest cases for students to see might be the 360-gon, since it has rotational symmetry at every degree. However, if necessary, a follow-up question might be, "What rotational symmetries does a regular triangle have?" This might help students focus on the fact that a regular polygon will have more than one angle of rotational symmetry—in fact, it will have all the multiples of a given number of degrees of rotational symmetry.

Focus on Student Thinking—*Continued*

As the students begin to find examples of regular polygons with 80-degree rotational symmetry, ask them, "Have you found them all? Are you sure that there might not be another one?"

By focusing on the factors of 80, many students are likely to see that the following eight regular polygons work. (More possibilities exist, however!)

A 9-gon, since it has 40-degree symmetry, also has 80-, 120-, 160-, ... degree rotational symmetries.

A 18-gon, since it has 20-degree symmetry, also has 40-, 60-, 80-, ... degree rotational symmetries.

Similarly, a 36-gon, 45-gon, 72-gon, 90-gon, 180-gon, and a 360-gon all have 80-degree rotational symmetry.

Some students might initially focus on these factors of 80 and be uncomfortable with (or just not think of) fractional degrees of rotational symmetry. For example, they might think that a 27-gon doesn't really work in the context of the task. However, asking whether it will fit onto itself if it is turned just the right amount might help, as might asking whether one can ever have a fractional number of degrees. They might then conclude:

A 27-gon has 13 1/3-degree symmetry, so it also has 26 2/3-, 40-, 53 1/3-, 80-, ... degree rotational symmetry.

Likewise, a 54-gon, 63-gon, ... will also work.

To help students find all the possibilities, you might ask, "Can you find a pattern in the regular polygons that we have found to have 80-degree rotational symmetry?" Students might see that in all of their solutions, the number of the polygons' sides is divisible by 9. Encourage them to try both examples and counterexamples to see whether this pattern holds. You should also encourage them to prove that their conjecture is correct.

To show why this pattern works, a student might argue as follows: A regular n-gon will have 80-degree rotational symmetry if $(360/n)$ divides evenly into 80. In other words, $80/(360/n)$ must be an integer, or in simplified form, $(2/9)n$ must be an integer. This implies that a regular n-gon will have 80-degree rotational symmetry. So, only multiples of 9 will work for the numbers of sides.

Assessment

For homework, you might have students write up a complete solution, explaining which regular polygons have 80-degree rotational symmetry and justifying that they have found all the possibilities.

Extensions

You could also ask students to find all regular polygons with other amounts of rotational symmetry to demonstrate that they understand the reasoning involved in solving the problem.

Resources

Common Core State Standards Initiative (CCSSI). *Common Core State Standards for Mathematics. Common Core State Standards (College- and Career-Readiness Standards and K–12 Standards in English Language Arts and Math)*. Washington, D.C.: National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010. http://www.corestandards.org.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

————. *Focus in High School Mathematics: Reasoning and Sense Making.* Reston, Va.: NCTM, 2009. Example 13, pp. 52–53.

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Student Activity Sheet

What regular polygons have 80-degree rotational symmetry? Explain your reasoning.