Ch 2..Implicit Differentiation

TODAY...

we will discover the reason and method for implicit differentiation

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Ch 2..Implicit Differentiation

Guidelines for Implicit Differentiation

1. Differentiate both sides of the equation with respect to x

2. Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side of the equation

3. Factor dy/dx out of the left side of the equation

4. solve for dy/dx by dividing both sides of the equation by the left hand factor that does not contain dy/dx

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implicit form... $xy = 1$

explicit form... $y = 1/x$

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note the similarities and differences of each form...

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dy/dx reads " the derivative of y with respect to x"

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dx/dx reads "the derivative of x with respect to x" . it also is equal to one (anything divided by itself is one!!

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Ch 2..Implicit Differentiation

sooo... in general.. d(whatever) /dx means the derivative of (whatever) with respect to x.

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we will be deriving many different variables in the long run, so take this lesson to build a foundation of understanding the general version of implicit differentiation.

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examples: $y = \frac{1}{x}$ ... find $\frac{dy}{dx}$

$$1\,dy = \frac{x(0\,d(1)) - 1(1\,dx)}{x^2}$$

$$\frac{dy}{dx} = \frac{-1}{x^2}\frac{dx}{dx}$$

$$\frac{dy}{dx} = \frac{-1}{x^2}$$

EXPLICIT

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implicit... $\left(y=\frac{1}{x} \rightarrow xy=1\right)$

$xy=1$ find dy/dx

$$\frac{x(1\,dy)}{dx}+\frac{y(1\,dx)}{dx}=\frac{0(dl)}{dx}$$

$$x\left(\frac{dy}{dx}\right)+y=0$$

$$-y \quad -y$$

$$\frac{x}{x}\left(\frac{dy}{dx}\right)=\frac{-y}{x}$$

$$\boxed{dy/dx=-y/x}$$

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Ch 2..Implicit Differentiation

Find dy/dx given: $y^3+y^2-5y-x^2=-4$

p.135 GUIDELINES FOR I.D.

$$3y^2\frac{dy}{dx}+2y\frac{dy}{dx}-5\frac{dy}{dx}-2x\frac{dx}{dx}=0$$

$$\frac{dy}{dx}\left(3y^2+2y-5\right)=\frac{2x}{(3y^2+2y-5)}$$

$$\frac{(3y^2+2y-5)}{(3y^2+2y-5)}$$

$$\boxed{\frac{dy}{dx}=\frac{2x}{(3y^2+2y-5)}}$$

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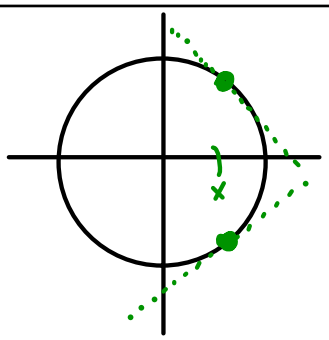
Ch 2..Implicit Differentiation

Unit Circle $x^2 + y^2 = 1$



Find dy/dx

$$2x\frac{dx}{dx} + 2y\frac{dy}{dx} = 0$$

$$2x + 2y\left(\frac{dy}{dx}\right) = 0$$

$$\frac{2y\left(\frac{dy}{dx}\right)}{2y} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = m = -\frac{1}{2} \Big/ \frac{\sqrt{3}}{2} = \frac{-1}{\sqrt{3}}$

$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right) = m = -\frac{1}{2} \Big/ -\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}$

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(ex) $x + y^2 = 1$ find dy/dx

$$x + y^2 = 1$$

$$\frac{1\,dx}{dx} + \frac{2y\,dy}{dx} = \frac{0}{dx}$$

$$1 + 2y\,dy/dx = 0$$

$$\frac{-1}{\quad} \qquad \frac{-1}{-1}$$

$$\frac{2y\,dy/dx}{2y} = \frac{-1}{2y}$$

$$\boxed{\frac{dy}{dx} = -1/2y}$$

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(ex) $3(\sin x \cos y) = 4$ find dy/dx

$-\sin x \sin y \frac{dy}{dx} + \cos y \cos x \frac{dx}{dx} = 0$

$-\cos y \cos x \quad -\cos y \cos x$

$\frac{dy}{dx} = \frac{-\cos y \cos x}{-\sin x \sin y}$

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Ch 2..Implicit Differentiation

ex Given $xy = 1$, find dy/dx

P.R. $x\frac{(1dy)}{dx} + y\frac{(1dx)}{dx} = \frac{0(d1)}{dx}$

$\frac{x}{x}\left(\frac{dy}{dx}\right) + y = 0$

$-y \quad\quad \frac{-y}{x}$

$$\frac{dy}{dx} = \frac{-y}{x}$$

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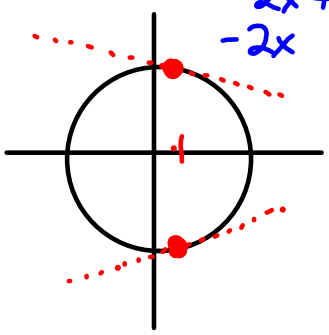
ex $x^2 + y^2 = 1$, find dy/dx ...

$2x\frac{(dx)}{dx} + 2y\frac{(dy)}{dx} = \frac{0}{dx}$

$2x + 2y\left(\frac{dy}{dx}\right) = 0$

$-2x \quad\quad \frac{-2x}{2y}$



$$\frac{dy}{dx} = \left(\frac{-x}{y}\right)$$

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Ch 2..Implicit Differentiation

Given $x^2 + xy + y^2 = 3$ Find dy/dx

$$2x\frac{dx}{dx} + x\frac{dy}{dx} + y\frac{dx}{dx} + 2y\frac{dy}{dx} = \frac{0}{dx}$$

$$\left(\frac{dy}{dx}\right)(x+2y) = -2x - y$$

$$\frac{x+2y}{x+2y}$$

$$\frac{dy}{dx} = \frac{-2x-y}{x+2y}$$

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Given: $3(\sin x \cos y) = 7$

find: dy/dx

$$\sin x\left(-\sin y\frac{dy}{dx}\right) + \cos y\left(\cos x\frac{dx}{dx}\right) = \frac{0}{dx}$$

$$\frac{dy}{dx}\left(\frac{-\sin x \sin y}{-\sin x \sin y}\right) = \frac{-\cos y \cos x}{-\sin x \sin y}$$

$$\frac{dy}{dx} = \frac{-\cos y \cos x}{-\sin x \sin y}$$

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Given: $x^3 + y^2 = 9$

Find: dy/dx

$$3x^2 \frac{dx}{dx} + 2y\frac{dy}{dx} = 0$$

$$3x^2 + 2y\left(\frac{dy}{dx}\right) = 0$$

$$-3x^2 \qquad\qquad -3x^2$$

$$\frac{2y\left(\frac{dy}{dx}\right)}{2y} = \frac{-3x^2}{2y}$$

$$\left(\frac{dy}{dx}\right) = -\frac{3x^2}{2y}$$

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