Ch 2 (Related Rates)

example ...

find the 'rate of Change' of Volume for a Cone.

$V = \frac{\pi}{3} r^2 h$ ← (Formula)

$dV = \frac{\pi}{3}(r^2 dh + h 2r dr)$

R.W. $\frac{dV}{dt} = \frac{\pi}{3}\left(r^2 \frac{dh}{dt} + 2hr \frac{dr}{dt}\right)$

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Ch 2 (Related Rates)

p.141 example 1

$y = x^2 + 3$

$\frac{dy}{dt} = 2x \frac{dx}{dt}$

- find dy/dt
- when $x = 1$
- $x = 1$, $dx/dt = 2$

$\frac{dy}{dt} = 2(1)(2) = 4$ units/time

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Ch 2 (Related Rates)

Ex #2 p.142

$A_o = \pi r^2$

$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(4)(1)$

$\frac{dt}{dt} = 25.132 \text{ ft}^2/\text{sec}$

- $\frac{dr}{dt} = 1 \text{ ft/sec}$
- $r = 4 \text{ ft}$

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Ch 2 (Related Rates)

$d = \sqrt{(y-y)^2 + (x-x)^2}$

$x^2 + h^2 = s^2$

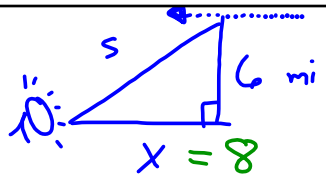
$2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 2s\frac{ds}{dt}$

$\frac{dx}{dt} = \frac{-2h\,dh + 2s\,ds}{2x}$

$dx/dt = \frac{-2(6)(0) + 2(10)(-400)}{2(8)}$

$dx/dt = -500 \text{ miles/hr}.$

$\therefore dx/dt = 500 \text{ mph}$



$\frac{ds}{dt} = -400 \text{ m/h}$

$s = 10 \text{ miles}$

find $\frac{dx}{dt}$

(*) SPEED IS ALWAYS POSITIVE

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Ch 2 (Related Rates)

p. 146 # 1a $y = \sqrt{x}$

RW $y = x^{1/2}$

$\frac{dy}{dt} = \frac{1}{2}x^{-1/2}\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{1}{2}(4)^{-1/2}(3)$

$\frac{dy}{dt} = \frac{3}{4}$

$\frac{dy}{dt} = \frac{1}{2\sqrt{4}}(3)$

$\frac{dy}{dt}$ when $x = 4$, $\frac{dx}{dt} = 3$

Shannon

Madison

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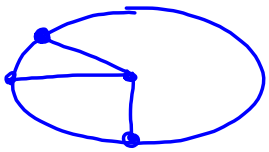
Ch 2 (Related Rates)

$y = x^2 + 1$ (x, y) $(0, 0)$



$d = \sqrt{(0 - (x^2+1))^2 + (0 - x)^2}$ "y"

$d = \sqrt{(-x^2-1)^2 + x^2}$

$d = \sqrt{x^4 + 2x^2 + 1 + x^2} = \sqrt{x^4 + 3x^2 + 1}$

$\frac{dd}{dt} = \frac{1}{2}\left(x^4 + 3x^2 + 1\right)^{-1/2}\left(4x^3 + 6x\right)\left(\frac{dx}{dt}\right)$ $= 2$

$\frac{dd}{dt} = \frac{4x^3 + 6x}{\sqrt{x^4 + 3x^2 + 1}}$ cm/sec

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Ch 2 (Related Rates)

ex

find the rate of Change of the volume of a Cylinder ...

$V = \pi r^2 h$

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + 2hr\frac{dr}{dt}\right)$$

differentiated with respect to time

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Ch 2 (Related Rates)

p.146 #6 $dy/dt = 2$

$y = \sqrt{x}$, find dx/dt, $x = 25$

$y = \sqrt{25}$ $y = 5$

$\frac{dy}{dt} = \frac{1}{2}x^{-1/2}\frac{dx}{dt} \Rightarrow 2 = \frac{1}{2}(25)^{-1/2}\frac{dx}{dt}$

$\frac{1}{2}(25)^{-1/2} = \frac{1}{2}\cdot\frac{1}{\sqrt{25}} = \frac{1}{10}$

$\frac{2}{\frac{1}{2}(5)^{-1}} = dx/dt$

$20 = dx/dt$

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Ch 2 (Related Rates)

2a

$y = x^2 - 3x$, $x = 3$, $dx/dt = 2$

$$\frac{dy}{dt} = 2x\frac{dx}{dt} - 3\frac{dx}{dt}$$

$$\frac{dy}{dt} = 2(3)(2) - 3(2)$$

$$\frac{dy}{dt} = 6$$

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Ch 2 (Related Rates)

p.143 ex # 3

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\frac{dr}{dt}\right)$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$4.5 = 4\pi(2^2)\,dr/dt$$

$$0.09 \text{ in/min} = dr/dt$$

$\frac{dV}{dt} = 4.5 \text{ in}^3/\text{min}$

$r = 2$ in

$\frac{dr}{dt} = ?$

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$4.5 = 4\pi(2^2)\,dr/dt$

$4.5\ in^3/min = \dfrac{4\pi(2\,in)^2}{4\pi(4\,in^2)}\dfrac{dr}{dt}$

$\dfrac{4.5\ in^3}{4\pi(4in^2)min} = \dfrac{4.5\,in^3}{16\pi\,in^2\,min}$

$\dfrac{4.5}{16\pi}\ in/min$

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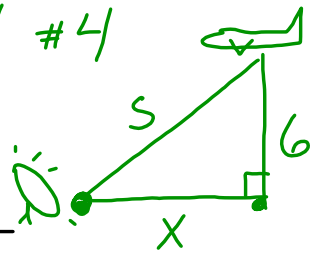
Ch 2 (Related Rates) p.144 #4

$a^2+b^2=c^2$

$x^2+6^2=10^2$

$x=8$



$s=\sqrt{(6)^2+(x)^2}$

$\dfrac{ds}{dt}=\dfrac{1}{2}(6^2+x^2)^{-1/2}(2x\,dx/dt)$

$-400=\dfrac{1x\,dx/dt}{(36+(x)^2)^{1/2}}$

$-400=\dfrac{8\,(dx/dt)}{10}$

speed = 500 mi/hr

when $s=10$ mi

$\dfrac{ds}{dt}=-400\ mi/hr$

$x=8$ mi

* speed must be positive

$-500=dx/dt$

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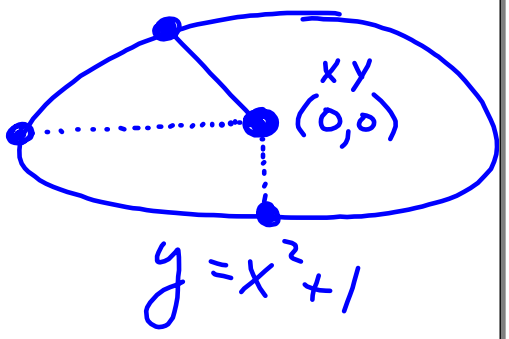
Ch 2 (Related Rates)



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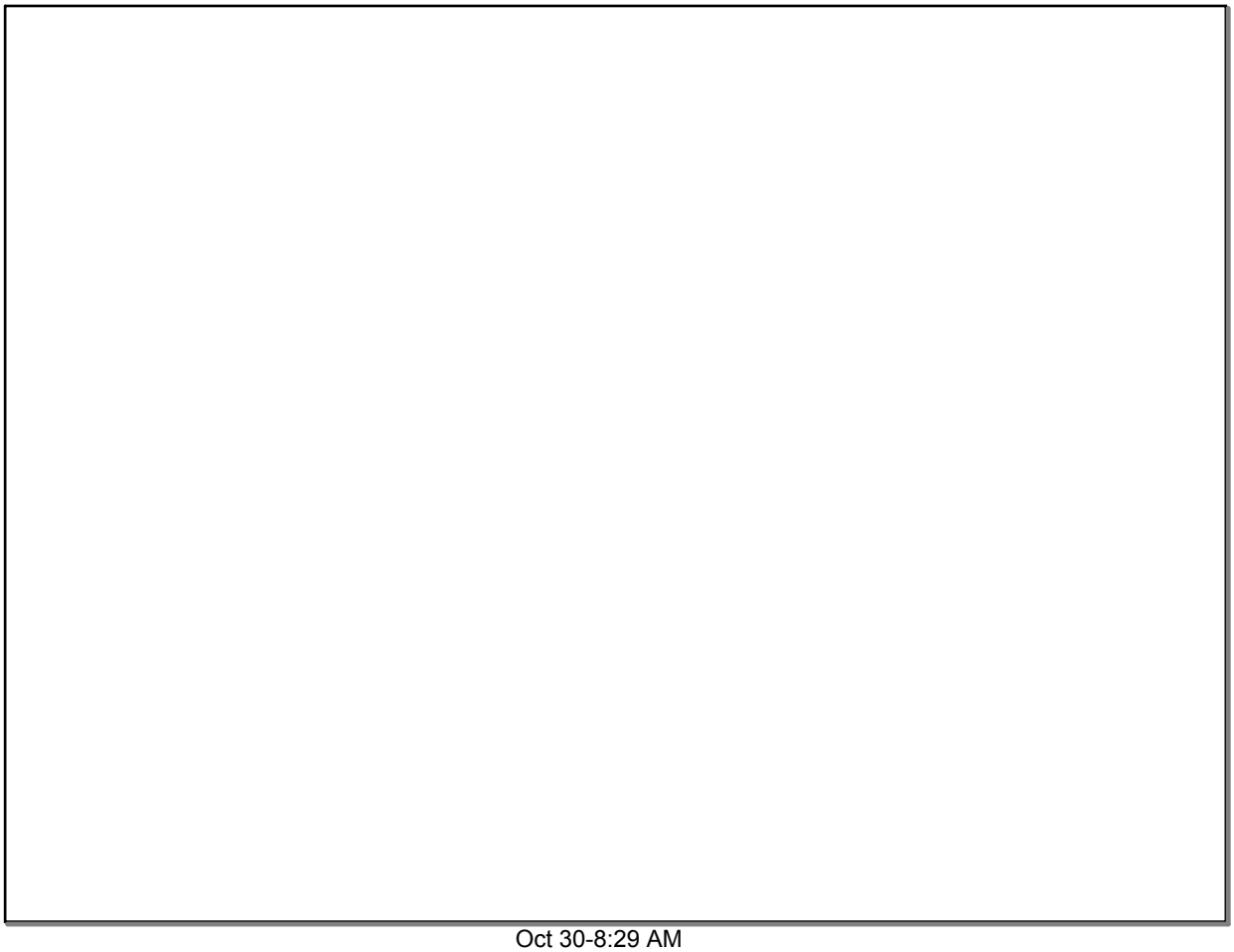

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